Temperature and heat

Unit 7

Contents

Section	Learning competencies
7.1 Temperature and heat (page 172)	• Explain the difference between heat and temperature. • Define the term thermal equilibrium.
7.2 Expansion of solids, liquids and gases (page 179)	• Describe the thermal expansion of solids and derive the expression for the linear and surface expansion of solids. • Find the relationship between the coefficient of linear, area and volume expansion and solve related problems. • Know applications of the thermal expansion of materials. • Distinguish between apparent and real expansion of a liquid and solve problems involving the expansion of liquids using $V = V_o \gamma \Delta T$. • Explain the abnormal expansion of water. • Compare the expansion of gases with the expansion of solids and liquids.
7.3 Quantity of heat, specific heat capacity and heat capacity (page 191)	• Describe the factors that affect the amount of heat absorbed or liberated by a body. • Define the terms specific heat capacity and heat capacity and calculate the amount of heat energy absorbed or liberated by a body using $Q = mc\Delta T$. • Calculate the heat capacity of a body. • Identify different units of heat energy. • Explain the significance of the high specific heat capacity of water. • Use the relationship heat lost = heat gained to solve problems involving heat exchange. • Describe the uses of a calorimeter.
7.4 Changes of state (page 199)	• Define the terms latent heat, latent heat of fusion and latent heat of vaporisation. • Solve problems involving change of state.

On a hot day our ice cream melts more quickly, but why? On a cold day we may need a coat and if its gets very cold it might even snow. Our perception of temperature is all relative; what's cold to us might be described very differently from a resident of northern Canada!

The concepts of heat and temperature are not just used in weather forecasting. The bread in an oven needs to be baked at just the right temperature, the wheels are fitted onto a train's axle using low temperatures in a technique called shrink fitting, and air conditioning and central heating systems only function due to our understanding of heat and temperature.

This unit looks at the meaning of the terms heat and temperature, the effects of different temperatures and some applications that rely on these phenomena.

KEY WORDS

atoms *the smallest parts of a chemical element that can take part in a reaction*

average kinetic energy *the mean kinetic energy of all the particles in a substance*

ions *an atom or group of atoms that has acquired an electrical charge by gaining or losing one or more electrons*

molecules *the smallest unit of a substance, consisting of a group of atoms, which retains the chemical and physical properties of the substance*

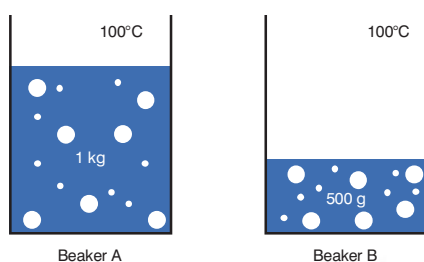**particles** *small units of matter such as atoms, ions and molecules*

By the end of this section you should be able to:

- Explain the difference between heat and temperature.
- Define the term thermal equilibrium.

What is heat?

When we cook food, we might say we are heating it up. The temperature of the food increases. It seems like heat and temperature are the same thing, but they are not!

We already know that matter is made up of moving **particles** (**molecules**, **atoms** and **ions**). In solids these particles are tightly bonded together and so they can only vibrate, whereas in fluids (liquids and gases) the particles can move around more freely.

Heat is one form of energy; it is therefore measured in joules and is a *scalar* quantity. Heat is a flow of energy from hotter regions to colder ones.

- $Q = \Sigma E_k + \Sigma U$

Q is the symbol used for heat energy. From Unit 4, E_k is kinetic energy and U is potential energy. Remember, Σ means sum of.

Imagine two beakers of boiling water. Beaker A contains 1 kg of boiling water and beaker B contains 0.5 kg of boiling water. They are both at the same temperature, 100 °C, but there are more particles in beaker A and so there is more energy contained within it than with beaker B.

Figure 7.1 Two beakers of boiling water

When we heat up a substance, we are transferring energy to the substance. This means one of two things could happen.

- **The particles of the substance gain kinetic energy and so move more rapidly.**

and/or

- **The bonds between the particles in the substance are broken and the potential energy of the particles increases. When this happens, the substance changes state.**

What is temperature?

Temperature is a measure of 'hotness'. The higher the temperature, the hotter the object. The complication is that 'hotter' may not mean more heat when comparing two objects.

The temperature of a substance is a measurement of the **average kinetic energy** of the particles within the substance. If the particles in a substance have a higher average kinetic energy then the object is at a higher temperature. That is to say if the particles are, on average, moving faster then the object is at a higher temperature. The water molecules in a glass of water at 50 °C are, on average, moving faster than those in a glass at 20 °C.
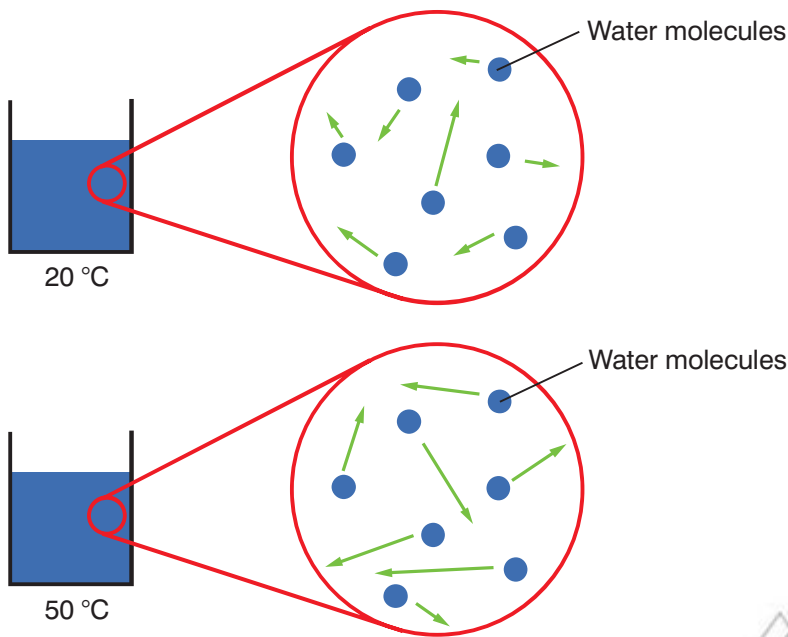

Figure 7.2 A hot day. But what is the difference between heat and temperature?

Figure 7.3 On average, the particles are moving faster if the object is at higher temperature.

10 kg of water at room temperature may contain more energy than a tiny metal spark from a sparkler. However, the spark is at higher temperature (maybe 500 °C compared with 25 °C). Temperature is a measure of the average kinetic energy of the particles; heat is the total thermal energy inside the substance (the total kinetic and potential energies added together). On average, the particles are moving faster in the spark; however, there are far more particles in the water, all with a kinetic energy and potential energy.

It is important to notice we use the average kinetic energy. In any substance some particles will be moving faster than others and so these particles will have more kinetic energy than the others.
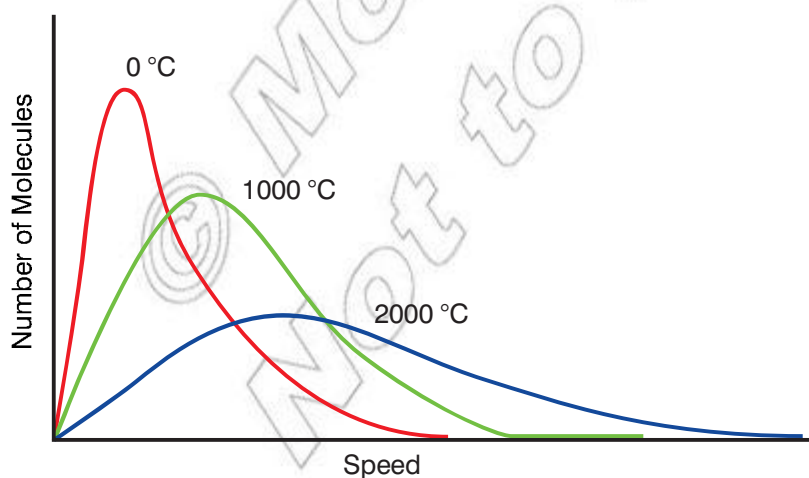

Figure 7.4 The water may contain more heat energy even though it is at a lower temperature than the spark.

Figure 7.5 This graph shows how many particles have a given speed at three different temperatures.

Think about this...

As the temperature of a body indicates the average kinetic energy of the particles, it *does not* depend on the number of particles present.

KEY WORDS

absolute zero *the temperature at which a substance has no thermal energy*

Celsius scale *a temperature scale where the freezing point of water is fixed at 0 degrees and the boiling point at 100 degrees C*

Kelvin scale *a temperature scale that uses absolute zero as one of its fixed points*
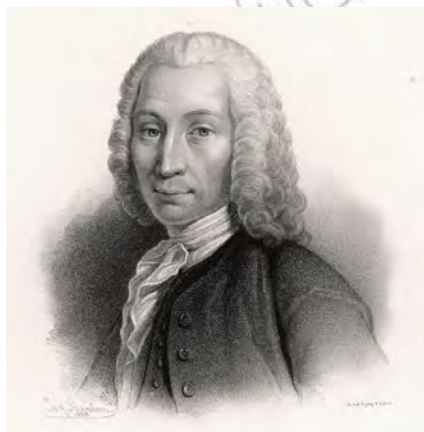
Figure 7.7 The Swedish astronomer Anders Celsius first proposed the Celsius scale in 1742.

If we look at Figure 7.5, we can see that at 0 °C, most particles have a relatively low speed and hence a relatively low kinetic energy – only a relatively small number of particles have a high speed and a high kinetic energy. As a substance is heated to a higher temperature, for example 1000 °C, the graph shows us that the average speed and consequently average kinetic energy of the particles is greater than at 0 °C.

Temperature scales

A range of temperature scales have been used in the past, though scientists now tend to deal with the **Kelvin scale** (K) or the **Celsius scale** (°C). The Fahrenheit scale (°F) is still used by the United States but is rarely used by the scientific community.

To design a temperature scale two fixed reference points have to be used. The scale is then based on these points with a certain number of jumps in between them.

For example, in the Celsius scale, the freezing point of water is given as 0 °C , with the second fixed point being the boiling point of water – the difference between the two fixed points is divided into 100 equal divisions so the boiling point of water is 100 °C.

Figure 7.6 The freezing point and boiling point of water were the two fixed points used on the Celsius scale.

The Kelvin scale uses **absolute zero** as one of its fixed points. This is the temperature at which a substance will have no thermal energy and it is not possible to get a lower temperature (0 K or –273.15 °C). The Kelvin scale has a units symbol of K; there is no degree symbol included.

The Kelvin and the Celsius scales are often used together as they have the same scale division. This means a *change* of 20 K is the same as a change of 20 °C.

The Kelvin scale may seem unusual as it uses fixed points that we are not familiar with but, importantly, temperatures measured in the Kelvin scale are directly proportional to the average kinetic energy of the particles present. For example, the particles in a block

of iron have on average twice as much kinetic energy at 200 K than at 100 K.

The Fahrenheit scale uses one fixed point as the temperature of an ice, water and ammonium chloride mixture (0 °F) – the second fixed point is normal body temperature (98 °F).

The diagram in Figure 7.8 shows how these temperature scales compare. We will use the Celsius and Kelvin scale in our calculations.
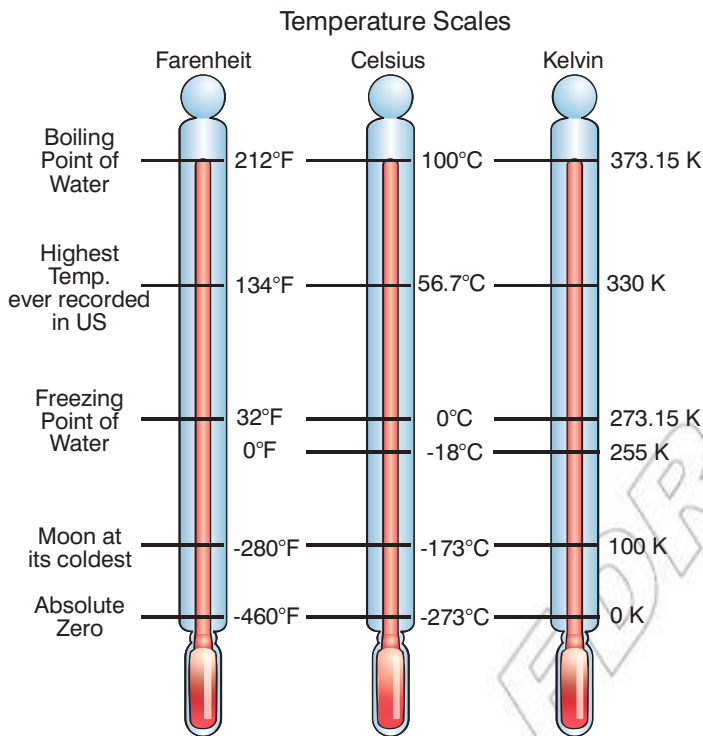
Temperature Scales

Figure 7.8 Comparing different temperature scales

A bouncing ball

If a ball is bounced repeatedly, it gains heat energy and its temperature increases. The photographs in Figure 7.9 show thermal images of a ball before and after the ball is bounced. The higher temperatures are shown progressively in red, orange, and yellow; green and blue indicate lower temperatures

Look at the temperature scales down the side of the photos – what temperature scale do you think is being used and where has the heat energy come from to increase the temperature of the ball?
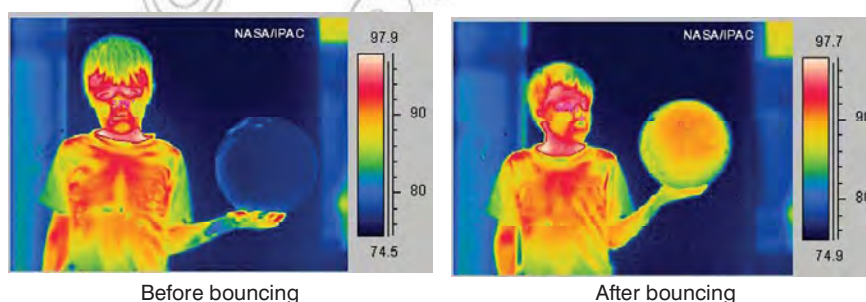

Before bouncing   After bouncing

Figure 7.9 Bouncing a ball will increase its temperature.

DID YOU KNOW?

The Celsius scale is named after the Swedish astronomer Anders Celsius. In 1742 he proposed the Celsius temperature scale, with one key difference. He set his lower fixed point (the freezing point of water) as 100 °C and the boiling point as 0 °C. This meant using his scale the number got smaller as the substance got hotter! The scale was reversed the year after he died.

Think about this...

At absolute zero a substance will have zero internal energy. What does this tell you about the kinetic energies and potential energies of the particles? Do you think it will be possible to reach absolute zero?

DID YOU KNOW?

The place that has the world's highest average temperature is Dalol, Ethiopia, in the Danakil Depression. The average temperature is an almost unbelievable 35 °C, or 308 K.

Activity 7.1: Temperature scales

How would you convert a temperature recorded on the Celsius scale into a temperature on the Kelvin scale (K) and vice versa?
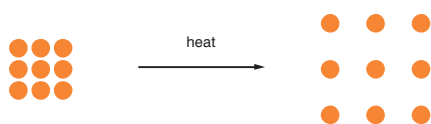
Figure 7.10 *As the particles vibrate more they spread out and so the substance expands.*

Figure 7.11 *A and B are in thermal contact. A is hotter than B; it has a higher temperature.*

What happens when a substance absorbs heat energy?

As a substance absorbs heat energy, the particles vibrate more (in a solid) or move faster (in a liquid or gas) as the heat energy is converted into the kinetic energy of the particles as the temperature rises. As the particles gain more energy, we can see that they move further apart from each other, which means the substance will **expand** (increase in size).

The diagram in Figure 7.10 shows how the particles in a solid move further apart as the solid is heated.

The expansion of substances on heating is called **thermal expansion**. This happens in solids, liquids and gases. We will deal with this in more detail in Section 7.3.

Thermal equilibrium

Heat energy flows from a hotter body to a colder body. Place your hand near an oven and you can feel the heat energy flowing into our hand. It feels hot! Place your hand inside a fridge and the heat energy flows from you into the fridge, it feels cold. An ice cube in boiling water will absorb heat energy from the hotter water, but the same ice cube in deep space will radiate heat energy to its surroundings.

When there is a movement of heat energy from a hotter object to a colder object, we say that the two objects are in **thermal contact**. Objects in thermal contact do not have to be in physical contact but they could be touching each other. So, we would say that the ice cube and the boiling water are in thermal contact with each other.

Imagine that we have two objects, A (at 90 °C) and B (at 50 °C). A and B are in thermal contact. There will be a *net* flow of heat energy will flow from A to B.

As heat energy is lost from A, the particles in A will slow down. They have, on average, less kinetic energy and so the temperature of A will decrease. The opposite happens at B. As B gains heat energy, the particles in B move faster, their average kinetic energy will increase and so the temperature of B rises.

This process of heat loss from A and heat gain by B will go on until A and B both reach the same temperature. At this point, **thermal equilibrium** is reached (heat loss from A will equal heat gained by B so that there is no net movement of heat energy between the two bodies).

- **If two bodies are in thermal equilibrium, they will also be at the same temperature.**

The details of how two bodies in thermal contact obtain thermal equilibrium are governed by the first and second laws of thermodynamics.

Think about this...

The concept of thermal equilibrium allows us to measure temperature. Imagine our object B is a thermometer; what temperature will the thermometer read when it is at thermal equilibrium with object A? Will this temperature be *exactly* the same as the original temperature of A?

First law of thermodynamics

The first law of thermodynamics has more than one form but all are really different ways of saying the same thing. It is essentially the **law of conservation of energy**; that is, that energy cannot be created or destroyed but can be transformed into other forms.

Imagine a gas that has a certain internal energy (the sum of the gas particles' kinetic and potential energy). The increase in internal energy of the gas, ΔU, will be equal to the heat energy it has gained, ΔQ, plus any work done on the gas, ΔW (for example if it is compressed).

- $\Delta U = \Delta Q + \Delta W$
- ΔU = change in internal energy in J
- ΔQ = heat energy *added to* system in J
- ΔW = work done *on* system in J

Notice in this case ΔU is the internal energy of the gas. Even though it is the sum of the kinetic and potential energies of the particles in the gas it is essentially a potential (stored) energy in the gas, hence the symbol U. All these terms are energies and so measured in joules.

We can see from the equation that if no work is done ($\Delta W = 0$), the heat energy we add to the object will equal the increase in internal energy. This means the temperature of the object will rise. In other words, energy has not been created or destroyed, just transformed into other forms.

We can also increase the internal energy by doing work on the substance. Imagine the gas inside a pump. If we rapidly compress the pump with our thumb over the end we are doing work **on** the gas inside it. In this case the work goes into increasing the internal energy of the gas. The gas gets hotter.

The equation may be used to calculate the change in internal energy, if there is work being done and there is a flow of heat into a substance.

For example, consider a gas that is being heated and compressed. There is a heat flow into the gas of 500 J and 200 J of work is done on the gas by compressing it. The change in internal energy is:

$\Delta U = \Delta Q + \Delta W$ *State principle or equation to be used (First law of thermodynamics)*

$\Delta U = 500\ \text{J} + 200\ \text{J}$ *Substitute in known values and complete calculation*

$\Delta U = 700\ \text{J}$ *Clearly state the answer with unit*

However, what if the object is hotter than its surroundings? Imagine a cup of tea. If you stir it really fast you might do 20 J of work on the tea. At the same time there has been a flow of heat from the tea to the surroundings of 100 J. What is the change in internal energy?

$\Delta U = \Delta Q + \Delta W$ *State principle or equation to be used (First law of thermodynamics)*

$\Delta U = -100\ \text{J} + 20\ \text{J}$ *Substitute in known values and complete calculation*

$\Delta U = -80\ \text{J}$ *Clearly state the answer with unit*

Figure 7.12 Heating or doing work on a gas increases its internal energy and so its temperature.

DID YOU KNOW?

There is also a zeroth law of thermodynamics, which states that if object A and B are individually in thermal equilibrium with another object C, objects A and B are also in thermal equilibrium with each other. This law has important implications – it means that if C is our thermometer, it will indicate the same temperature for both objects A and B.

KEY WORDS

expand *to increase in size*

thermal contact *when there is a movement of heat energy between a hotter and a colder object*

thermal expansion *the increase in size of a substance as a result of heating*

thermal equilibrium *situation where there is no net movement of heat energy between bodies*

Figure 7.13 Doing work on a cup of tea as heat flows from the cup to the surroundings.

Figure 7.14 There is a net energy flow from hotter to colder objects.
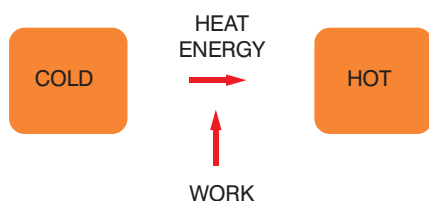

Figure 7.15 Energy will flow from colder to hotter objects if work is done on the system.

Figure 7.16 In a fridge heat flows from inside the fridge to its surroundings even though it is at a lower temperature.

In this case the tea has lost 100 J to the surrounds so $\Delta Q = -100$ J. The overall change in internal energy is –80 J, so the tea's temperature will fall. Theoretically it is possible to stir it fast enough to keep the tea at the same temperature. In which case $\Delta U = 0$ J and so $\Delta Q = -\Delta W$, but you would have to stir it very fast!

Second law of thermodynamics

The second law of thermodynamics concerns the direction of heat flow between two bodies. Usually, as we have seen when we looked at thermal equilibrium, heat energy flows spontaneously from hotter objects to colder objects. The second law of thermodynamics might be expressed as:

- **Heat generally cannot flow spontaneously from a material at lower temperature to a material at higher temperature.**

Heat energy will not flow from a colder object to a hotter one spontaneously unless work is done. Energy must be used to reverse the usual flow of heat energy. This principle is used in refrigerators, freezers and air conditioning units. The contents of a fridge are cooled by a liquid evaporating, but work has to be put in so as to condense the gas for further use.

Summary

In this section you have learnt that:

- Heat is energy transferred from hotter regions to cooler ones.

- The temperature of a substance is an indication of the average kinetic energy of the particles and the Celsius and Kelvin scales are both temperature scales.

- On heating, the particles of a substance move faster and move further apart so that a substance expands on heating.

- The first law of thermodynamics states that during heat transfer processes, energy cannot be created or destroyed.

- The second law of thermodynamics states that heat energy will flow from hot objects to colder objects and that if work is put in, heat energy can be removed from a cold object.

- When bodies are in thermal contact, heat energy flows from hot objects to cold objects until thermal equilibrium is reached and the bodies are at the same temperature.

Review questions

1. What will be the key difference in the energy of the particles in iron at 250 K and 500 K?

2. Explain why a solid expands on heating.

3. Describe what happens, in terms of the movement of heat energy, when a hot object is in thermal contact with a cold object. Explain how this process could be reversed.

4. Convert the following temperatures to the Kelvin scale:

   a) –273.15 °C

   b) 0.0 °C

   c) 1000 °C.

7.2 Expansion of solids, liquids and gases

By the end of this section you should be able to:

- Describe the thermal expansion of solids.
- Derive the expression for the linear and surface expansion of solids.
- Find the relationship between the coefficient of linear, area and volume expansion and solve related problems.
- Know applications of the thermal expansion of materials.
- Distinguish between apparent and real expansion of a liquid.
- Solve problems involving the expansion of liquids using $V = V_0\gamma\Delta T$.
- Explain the abnormal expansion of water.
- Compare the expansion of gases with the expansion of solids and liquids.

KEY WORDS

substance *material or matter*

The expansion of solids

We have already seen that when a solid is heated, its particles move further apart and hence the solid expands (increases in size). The ball and ring experiment shown in Figure 7.17 is a good demonstration of the expansion of a solid.
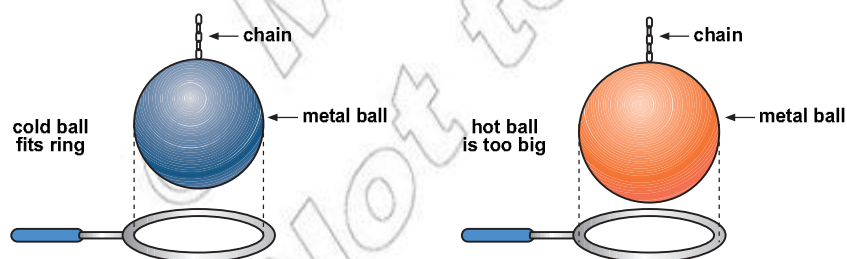

Figure 7.17 The hoop and ball experiment.

The cold metal ball easily passes through the ring. After heating, the metal ball expands and it is no longer able to pass through the ring.

How much a solid expands on heating will depends on the **substance** and how much its temperature increases.

Figure 7.18 When Concorde flew at over twice the speed of sound, it got so hot that it increased its 61.6 m length by about 30 cm.
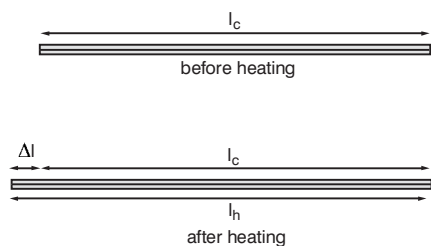
Figure 7.19 *Linear expansion of a narrow metal rod.*

Linear expansion of solids

When a metal rod is heated it expands and increases in length. This expansion is referred to as **linear expansion**. The diagram in Figure 7.19 shows a metal rod, of length l_c (measured in metres), before and after heating.

The rod's temperature has increased by ΔT. It increases in length on heating; the increase in length, Δl, is the difference between the length before heating, l_c, and the length after heating, l_h. This could be written as:

- $\Delta l = l_h - l_c$

So, for an increase in temperature of ΔT, the fractional increase in length $= \Delta l / l_c$. If a 50 cm rod expanded by 2 cm the fractional expansion would be:

- fractional increase $= \Delta l / l_c$
- fractional increase $= 2/50$
- fractional increase $= 1/25$ or 0.04

The fractional increase in length per unit of temperature (°C or K) increase is given the symbol α. It is found by dividing the fractional increase by the increase in temperature, ΔT.

- $\alpha = \Delta l / l_c / \Delta T$

Which is the same as:

- $\alpha = \Delta l / l_c \Delta T$

α is also known as the **coefficient of linear expansion** *for the solid.* It represents the increase in the length of a 1 m rod of a given substance when its temperature increases by 1 K. It is measured in /K or K^{-1}.

So, the increase in length of a heated rod, Δl, can be found by rearranging the above equation.

- $\Delta l = \alpha l_c \Delta T$

The values for the linear expansion coefficient of some solids are shown in Table 7.1.

Table 7.1 *The linear expansion coefficients of some solids*

Substance	Linear expansion coefficient ($\times 10^{-5}$ K^{-1})
aluminium	2.3
copper	1.7
brass	1.9
iron	1.1
concrete	1.2

This means that a 1 m iron rod will expand by 1.1×10^{-5} m for every 1 K rise in temperature. With these values, we can now calculate the increase in length of a material.

Worked example

Calculate the increase in length of a 50 cm brass rod that is heated from 25 °C to 70 °C.

$\Delta l = \alpha \, l_c \Delta T$ *Express Δl in terms of known factors*

In this case $l_c = 0.50$ m and $\Delta T = 70$ °C – 25 °C = 45 °C

$\Delta l = 1.9 \times 10^{-5}$ K$^{-1} \times 0.50$ m $\times 45$ °C *Substitute in known values and complete calculation*

$\Delta l = 4.3 \times 10^{-4}$ m *Clearly state the answer with unit*

Calculate the length of an iron rail at 400 °C, when it is exactly 100 m long at 200 °C.

$\Delta l = \alpha \, l_c \Delta T$ *Express Δl in terms of known factors*

In this case $l_c = 100$ m and $\Delta T = 400$ °C – 200 °C = 200 °C

- $\Delta l = 1.1 \times 10^{-5}$ K$^{-1} \times 100$ m $\times 200$ °C *Substitute in known values and complete calculation*

- $\Delta l = 0.22$ m *Clearly state the answer with unit*

Therefore, length of rail at 400 °C = 100 m + 0.22 m = 100.22 m
Ensure new length is calculated not just left as Δl

Activity 7.2: Expansion calculations

Calculate the increase in length of a 27 cm brass rod that is heated from 10 °C to 100 °C. $\alpha_{brass} = 1.9 \times 10^{-5}$ K^{-1}.

Calculate the length of a concrete section of a bridge at 45.00 °C, when it is 25.000 m long at 18.00 °C. $\alpha_{concrete} = 2.2 \times 10^{-5}$ K^{-1}.

Think about this...

Why do you think that it is safe to build a bridge made out of concrete reinforced with iron?

Figure 7.20 It is very important to consider the surrounding temperature and temperature variations when laying train tracks.

Surface (area) expansion of solids

In the examples we have looked at in linear expansion, the sample has been long in comparison to its height and width, so that the only significant expansion is in length. In practice, many objects are not long and thin and we need to develop a strategy to deal with these objects. We will start by looking at the expansion, in two dimensions, of a metal plate.

before heating
surface area = A_c

after heating
surface area = A_h

Figure 7.21 Two-dimensional expansion of an object

As the plate is heated to cause an increase in temperature, ΔT, it expands in width and height such that the surface area when heated, A_h, is larger than the original surface area, A_c. So:
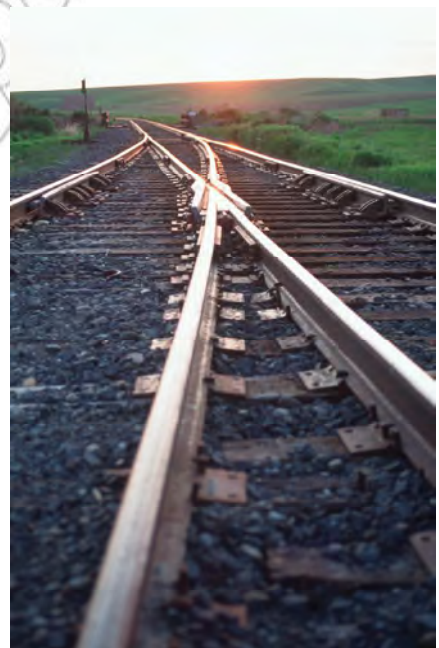
- $\Delta A = A_h - A_c$

before heating
surface area = $A_c = l_c^2$

after heating
surface area = $A_h = l_h^2$

$\alpha l_c \Delta T$

l_c

$l_h = l_c + \alpha l_c \Delta T$

l_c

l_c $\alpha l_c \Delta T$

$l_h = l_c + \alpha l_c \Delta T$

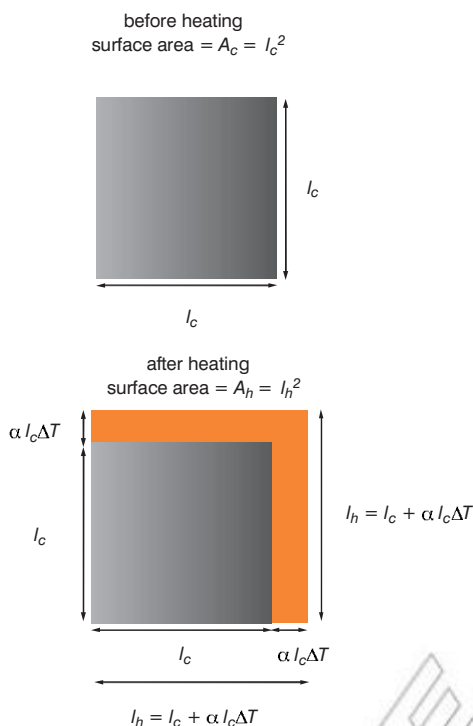*Figure 7.22 Relating linear expansion to surface expansion*

The fractional increase in surface area, β, per unit rise in temperature (°C or K) is given by:

- $\beta = \Delta A / A_c \Delta T$
- $\Delta A = \beta A_c \Delta T$

What is the relationship between α and β for a given substance?

We will start by recalling what β, the surface expansion coefficient, means and rewriting the expression:

- $\beta = \Delta A / A_c \Delta T$
- $\beta = A_h - A_c / A_c \Delta T$

Therefore:

- $\beta A_c \Delta T = A_h - A_c$

Making A_h the subject of the equation:

- $A_h = \beta A_c \Delta T + A_c$

which, after simplifying, gives:

- $A_h = A_c(1 + \beta\Delta T)$ – this expression will be of use later.

We will now write another expression for A_h, in terms of α, the linear expansion coefficient. If a square body of length l_c is heated such that its temperature increases by ΔT, each side increases in length, Δl, by $\alpha l_c \Delta T$ (see the section on linear expansion).

Consequently, the surface area of the heated body, is give by $A_h = l_h^2$.

l_h, the length of each side of the heated body is related to l_c:

- $l_h = l_c + \alpha l_c \Delta T$

Consequently,

- $A_h = l_h^2 = (l_c + \alpha l_c \Delta T)^2 = l_c^2(1 + \alpha\Delta T)^2 = l_c^2(1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$
- $A_h = l_c^2(1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$

We can further simplify this last form of the expression:

- $l_c^2 = A_c$
- $A_h = A_c(1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$

As α is a very small number, $\alpha^2\Delta T^2$ will be very small compared to $2\alpha\Delta T$ and so we will make an approximation and not include this small term in the final expression. In other words, $\alpha^2\Delta T^2$ is approximately zero, so:

- $A_h = A_c(1 + 2\alpha\Delta T)$

We can now compare this expression with the one we obtained earlier in terms of β:

- $A_h = A_c(1 + \beta\Delta T)$

Now we can see that $\beta\Delta T = 2\alpha\Delta T$ and therefore $\beta = 2\alpha$.

Remember, that this is an approximation but a very good one. We do not find tables of β values for substances as they are obtained from α values using β = 2α.

Volume expansion of solids

We now need to consider the expansion of a solid in three dimensions, where the length, breadth and height of the substance all increase on heating.

As the block is heated to cause an increase in temperature, ΔT, it expands in width, height and breadth such that the volume when heated, V_h, is larger than the original volume, V_c.

So:

- $\Delta V = V_h - V_c$

The fractional increase in volume, γ, per unit rise temperature (°C or K) is given by:

- $\gamma = \Delta V / V_c \Delta T$

- $\Delta V = \gamma V_c \Delta T$

What is the relationship between α and γ for a given substance?

We will start by recalling what γ, the volume expansion coefficient, means and rewriting the expression:

- $\gamma = \Delta V / V_c \Delta T$

Therefore:

- $\gamma = V_h - V_c / V_c \Delta T$

Making V_h the subject of the equation:

- $V_h = \gamma V_c \Delta T + V_c$

which, after simplifying, gives:

- $V_h = V_c(1 + \gamma \Delta T)$ – this expression will be of use later.

We will now write another expression for V_h, in terms of α, the linear expansion coefficient. If a cube, of length l_c, is heated such that its temperature increases by ΔT, each side increases in length, Δl, by $\alpha l_c \Delta T$ (see the section on linear expansion).

Figure 7.23 Three-dimensional expansion of an object

before heating
volume = V_c

after heating
volume = V_h

Activity 7.4: Volume expansion

Calculate the increase in the volume of an aluminium block with a volume of 0.008 m³ at 25.00 °C, when it is heated to a temperature of 90.00 °C. $\gamma_{aluminium}$ = 6.9 × 10⁻⁵ K⁻¹.

before heating
volume = $V_h = l_h^3$

after heating
volume = $V_h = l_h^3 = (l_c + \alpha l_c \Delta T)^3$

$l_h = l_c + \alpha l_c \Delta T$

$l_h = l_c + \alpha l_c \Delta T$

Figure 7.24 Relating linear expansion to volume expansion

Consequently, the volume of the heated body, is give by $V_h = l_h^3$

l_h, the length of each side of the heated body is related to l_c:

- $l_h = l_c + \alpha l_c \Delta T$

Consequently,

- $V_h = l_h^3 = (l_c + \alpha l_c \Delta T)^3 = l_c^3 (1 + \alpha \Delta T)^3 = l_c^3 (1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$

- $V_h = l_c^3 (1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$

We can further simplify this last form of the expression:

- $l_c^3 = V_c$

- $V_h = V_c (1 + 3\alpha \Delta T + 3\alpha^2 \Delta T^2 + \alpha^3 \Delta T^3)$

As α is a very small number, $\alpha^2 \Delta T^2$ and $\alpha^3 \Delta T^3$ will be very small compared to $3\alpha \Delta T$, and so we will make an approximation and not include these small terms in the final expression. In other words, $\alpha^2 \Delta T^2$ and $\alpha^3 \Delta T^3$ are approximately zero, so:

- $V_h = V_c (1 + 3\alpha \Delta T)$

We can now compare this expression with the one we obtained earlier in terms of γ:

- $V_h = V_c (1 + \gamma \Delta T)$

Now we can see that $\gamma \Delta T = 3\alpha \Delta T$ and therefore $\gamma = 3\alpha$.

Remember, that this is also approximation. Once again we do not find tables of γ values for substances as they are obtained from α values using $\gamma = 3\alpha$.
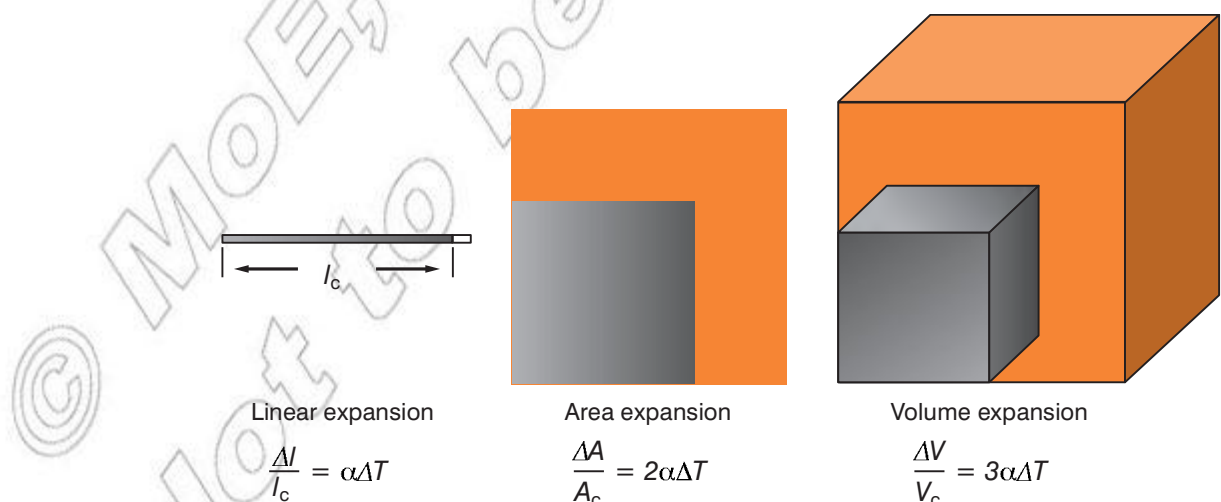
1D, 2D and 3D expansion summary

Figure 7.25 1D, 2D and 3D expansion summary in terms of α

It often helps to consider α as one-dimensional (1D), β as 2D (so 2α) and γ as 3D (so 3α).

Problems and applications of thermal expansion

The **thermal expansion** of objects can be a problem. Engineers have to allow for the expansion of concrete and iron on a hot day when building a bridge by constructing an expansion gap to allow for the expanding materials. Railway tracks also have expansion gaps and sections of pipelines in hot countries are linked by flexible pipe, which can accommodate the expanding pipe.

An open expansion joint on a bridge

Railway line

Figure 7.26 *Engineers must consider thermal expansion in a range of contexts.*

We can also take advantage of the expansion of materials and put them to good effect. In hot riveting, a hot steel rivet is used to join two metal sheets. Whilst still hot, the rivet is hammered to give a tight joint. As the rivet cools it contracts and makes the joint between the two metal sheets even tighter.
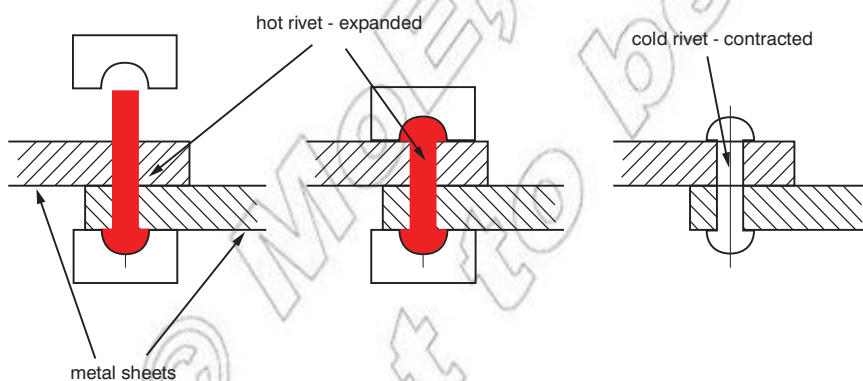

Figure 7.27 *Hot riveting uses the contraction of metals to make tighter connections.*

The bimetallic strip

A **bimetallic strip** is made out of two metals, for example iron and brass bonded together. The coefficient of linear expansion (α) of iron (1.1×10^{-5} K^{-1}) is less than that of brass (1.9×10^{-5} K^{-1}). When the strip is heated, the brass expands more than the iron and the strip bends.

Think about this...

How could you tell that the photo of the expansion joint on a bridge was taken on a cold day?

KEY WORDS

bimetallic strip *a strip made of two different metals bonded together along their length*
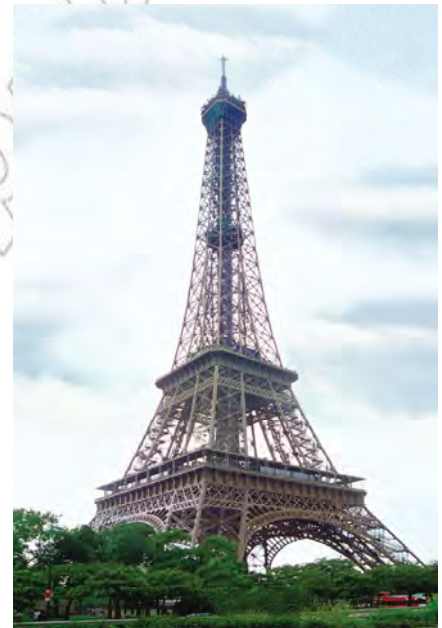

Figure 7.28 *The Eiffel Tower in Paris and the Sydney Harbour Bridge were constructed using hot riveting.*
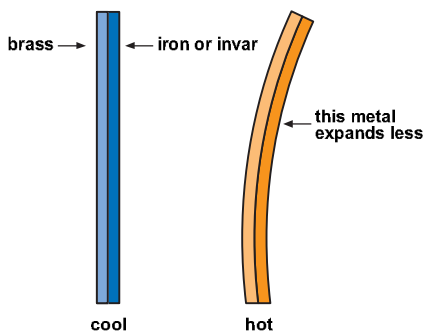
brass → ← iron or invar

← this metal expands less

cool   hot

Figure 7.29 The bimetallic strip

The bimetallic strip is used in a thermostat. This is a switch for an electric circuit that turns on and off according to the temperature.

brass

Invar
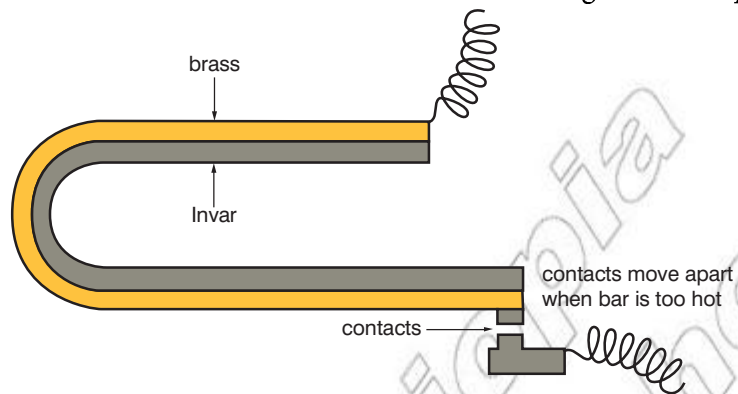
contacts move apart when bar is too hot

contacts

Figure 7.30 A bimetallic strip used as part of a thermostat

When the temperature rises, the brass section of strip expands faster than the iron and the strip bends so as to break the contact. As the temperature drops, the strip contracts, the contacts close and the circuit is restored. We can use this arrangement to switch on and off heating circuits in buildings and cookers, for example – when the desired temperature is reached, heating stops and it will not start again until the temperature has dropped.

Liquid in glass thermometers

Mercury in glass and alcohol in glass thermometers use the expansion of a liquid up a narrow glass tube. The higher the temperature, the more the mercury or alcohol expands and the further the liquids move up the capillary tube. As we have already seen, the Celsius scale uses two fixed points. We can calibrate a thermometer for the Celsius scale using the following method.

a) Place the bulb of an ungraduated thermometer in crushed ice – mark the level of the liquid (alcohol or mercury) when it stops moving. This is the first fixed point.

b) Place the bulb of the ungraduated thermometer in steam from boiling water. Mark the level of the liquid (alcohol or mercury) when it stops moving. This is the second fixed point.

c) Divide the distance between the two fixed points into 100 equal divisions – the first fixed point is at 0 °C and the second 100 °C.

> **KEY WORDS**
>
> **real expansion** *the actual increase in size of a substance*
>
> **apparent expansion** *the observed increase in size of a substance, which may be affected by the expansion of its container*

> **DID YOU KNOW?**
>
> The word thermometer comes from the Greek "thermo", which means warm, and "meter", to measure.

centigrade thermometer

lower fixed point

melting ice
filter funnel

beaker

water

centigrade thermometer

upper fixed point

mercury

steam

flask
boiling water

Figure 7.31 Calibrating a thermometer

Expansion of liquids

Liquids require a container and consequently it only really makes sense to discuss the volume expansion of a liquid. Liquids will tend to expand more than solids for a given increase in temperature but volume expansion coefficients can also apply to liquids.

- $\Delta V = \gamma V_c \Delta T$

  ΔV = change in volume.

  ΔT = change in temperature.

  V_c = starting volume.

  γ = volume expansion coefficient.

The expansion of liquids is made more complex, however, by the need for a container. On warming, the container itself will also expand. If you ask most people to predict what they will see when the apparatus below is heated, they will suggest that the water level will rise up the narrow glass tube as it expands.

The water level will indeed rise up the glass tube, but not before it first drops slightly! As the flask is heated, the glass it is made from expands and so the water level drops until the water itself warms up and expands. The expansion of a vessel holding a liquid means the actual or **real expansion** of the liquid is not actually observed. Instead, only an **apparent expansion** of the liquid is observed. Consequently, the extent to which we see the liquid expand (the apparent expansion) is less than its actual expansion (real expansion). It is possible to allow for the expansion of the vessel in calculations:

- $\gamma_{real} = \gamma_{apparent} + \gamma_{vessel}$

so:

- $\gamma_{apparent} = \gamma_{real} - \gamma_{vessel}$

We will use this relationship to calculate the real and apparent expansion of 1000 cm³ of water when it is warmed from 20 °C to 80 °C.

$\gamma_{glass} = 9.90 \times 10^{-6}\,K^{-1}$. $\gamma_{water} = 2.07 \times 10^{-4}\,K^{-1}$. $\Delta T = 80\,°C - 20\,°C = 60\,°C$.

- real expansion $= \Delta V_{real}$

$\Delta V_{real} = \gamma_{real} V_c \Delta T$ *State relationship to be used*

$\Delta V_{real} = 2.07 \times 10^{-4} \times 0.001\ m^3 \times 60\,°C$ *Substitute in known values and complete calculation*

$\Delta V_{real} = 1.24 \times 10^{-5}\ m^3$ *Clearly state the answer with unit*

- apparent expansion $= \Delta V_{apparent}$

$\gamma_{apparent} = \gamma_{real} - \gamma_{vessel}$ *State relationship to be used*

$\gamma_{apparent} = 2.07 \times 10^{-4}\,K^{-1} - 9.90 \times 10^{-6}\,K^{-1} = 1.97 \times 10^{-4}\,K^{-1}$ *Substitute in known values and complete calculation*

So:

$\Delta V_{apparent} = \gamma_{apparent} V_c \Delta T$ *State relationship to be used*
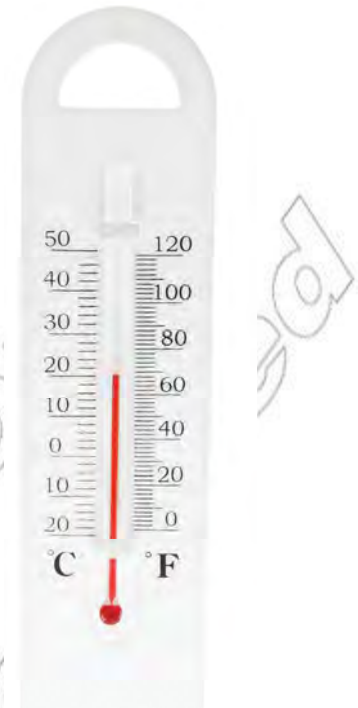

Figure 7.32 A thermometer is a simple yet very useful piece of equipment.

Activity 7.5: Expansion of a liquid

Calculate the increase in the volume of 0.0025 m³ of mercury at 5.00 °C, when it is heated to a temperature of 55 .00 °C. $\gamma_{mercury} = 1.8 \times 10^{-4}\,K^{-1}$.
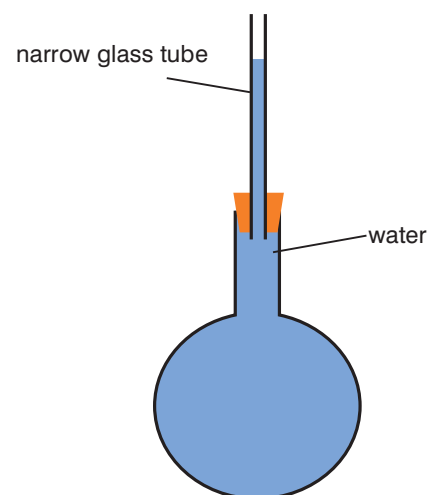

Figure 7.33 What will happen to the liquid if this object is heated?

$\Delta V_{\text{apparent}} = 1.97 \times 10^{-4}\ \text{K}^{-1} \times 0.001\ \text{m}^3 \times 60\ °\text{C}$ *Substitute in known values and complete calculation*

$\Delta V_{\text{apparent}} = 1.18 \times 10^{-5}\ \text{m}^3$ *Clearly state the answer with unit*

This calculation demonstrates that the real expansion of a liquid is greater than the apparent expansion.

Expansion of solids, liquids and gases

For a given change in temperature, ΔT, liquids will tend to expand significantly more than solids. This difference is clear when we compare values of coefficients of volume expansion for solids and liquids. We can see that $\gamma_{\text{liquid}} > \gamma_{\text{solid}}$.

Table 7.2 Volume expansion coefficient of solids and liquids

Substance (solid)	Volume expansion coefficient (γ) ($\times 10^{-5}\ \text{K}^{-1}$)	Substance (liquid)	Volume expansion coefficient (γ) ($\times 10^{-5}\ \text{K}^{-1}$)
aluminium	6.9	petrol	95.0
copper	5.1	ethanol	75.0
brass	5.7	water	21.0
iron	3.3	mercury	18.0

You will notice that there are no volume expansion coefficients for gases. This is because the volume of a gas is dictated by a number of factors. The temperature is certainly one of these, but we also have to consider the pressure and the amount (number of moles) of gas present. The relationship between the volume of a gas and its temperature can be shown using the ideal gas equation:

- $pV = nRT$

  p = pressure of gas in Pa

  V = volume of gas in m^3

  n = number of moles of gas in mol

  R = universal gas constant (8.314 J/K/mol)

  T = absolute temperature in K

You will notice that there is no constant in the equation relating to the nature of the gas. The equation applies to all "ideal gases" and is a good approximation for most gases. If the pressure and the amount of gas are constant, we notice that the volume is proportional to the absolute temperature:

- $V \propto T$

So, if the absolute temperature of a given quantity of gas is doubled at constant pressure, the volume doubles!

Gases will consequently tend to expand more for a given temperature rise than liquids, which in turn expand more than solids.

The unusual behaviour of water

In most cases when a liquid is frozen, the solid formed will have a lower volume than the initial liquid. We can explain this in terms of kinetic theory.

In a solid, the particles are closer together than they are in a liquid. Hence, for a given mass of substance, the solid usually has a lower volume and a higher density than its liquid state. Water though is an exception. The graph in Figure 7.34 shows that water has a maximum density at just under 4 °C.

Let's look at this graph in a little more detail. As the water cools below 10 °C, the water behaves as expected at first. It increases in density as its particles lose energy and move closer together. Then, at just below 4 °C, the density starts to decrease. A drop in density can only mean that the water molecules are further apart in water at 2 °C than they are at 8 °C. Why is this?

To understand this unusual observation, we need to know more about the forces acting between water molecules. Water has the chemical formula H_2O; it is composed of two hydrogen atoms and one oxygen atom. Liquid water has a very high boiling point for its molecular size because strong intermolecular forces (hydrogen bonds) form between the oxygen atom in one molecule and a hydrogen atom in another. To boil water, these strong hydrogen bonds have to be broken.

Normally in liquid water, each water molecule forms one hydrogen bond to another water molecule. As the temperature of water drops towards 4 °C though, the molecules are closer together and each molecule begins to form a second hydrogen bond with another water molecule. In order for this second hydrogen bond to form, the molecules now need to be in very exact relative positions and the molecules end up moving further apart to allow this second bond to form. This gives water below 4 °C and ice a more open molecular structure than warmer water.
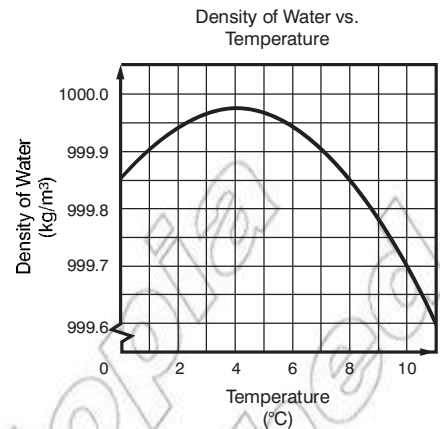

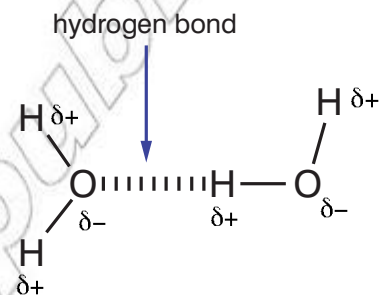**Figure 7.34** *How the density of water changes with temperature.*

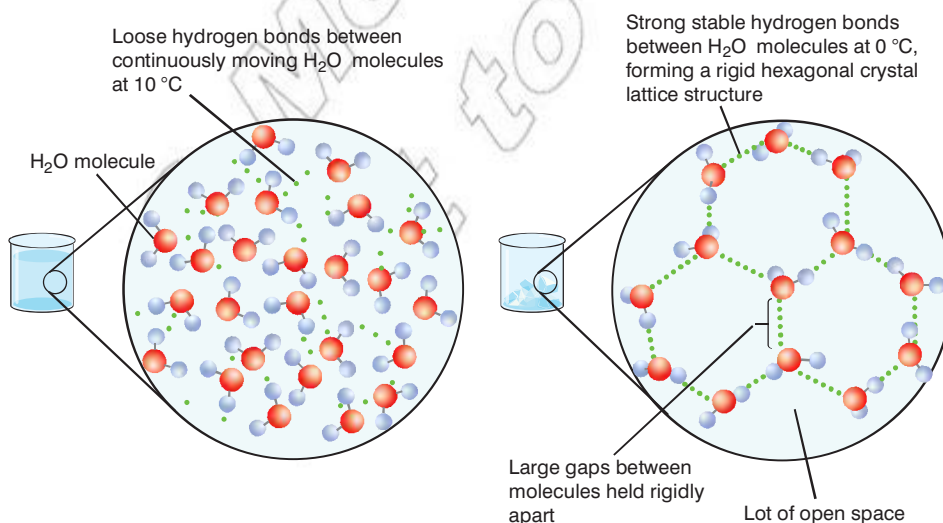**Figure 7.35** *A hydrogen bond between water molecules*

Figure 7.36 *The molecules are further apart in frozen water than in water at 4 °C.*

So, with a more open structure, ice and cold water below 4 °C have a lower density and a higher volume than warmer water. This explains why ice will float on water. The expansion of water on freezing can cause other problems though. When it gets cold, water in pipes can freeze, expand and then break the pipe!

Summary

In this section you have learnt that:

- The thermal expansion of solids can be explained in terms of the increasing distance between particles that occurs on warming.

- The expression for the linear expansion of solids is $\Delta l = \alpha l_c \Delta T$ – we can use this to find by how much the length of a sample expands.

- The expression for the surface area expansion of solids is $\Delta A = \beta A_c \Delta T$ – we can use this to find by how much the surface area of a sample expands.

- The expression for the volume expansion of solids is $\Delta V = \gamma V_c \Delta T$ – we can use this to find by how much the volume of a sample expands.

- The relationship between the coefficient of linear (α), area (β) and volume (γ) expansion is as follows: $\beta = 2\alpha$, $\gamma = 3\alpha$.

- The applications of thermal expansion include the bimetallic strip in thermostats, hot riveting and thermometers.

- The real expansion of a liquid is less than the apparent expansion as the vessel holding the liquid also expands.

- The abnormal expansion of water can be explained in terms of its more open molecular structure below 4 °C.

- Gases expand more than both solids and liquids for a given rise in temperature.

Review questions

1. Explain why solids expand on heating.

2. Calculate the increase in length of a 2 m brass rod that is heated from 0 °C to 150 °C. $\alpha_{brass} = 1.9 \times 10^{-5}$ K^{-1}.

3. Calculate the surface area of an iron drain cover with a surface area of 0.67 m^2 at 10 °C, when it is heated to a temperature of 105 °C. $\beta_{iron} = 2.2 \times 10^{-5}$ K^{-1}.

4. Show that, for a given material, the surface expansion coefficient (β) is about twice the linear expansion coefficient (α).

5. Calculate the increase in the volume of a 0.1 m^3 sample of water at 10.00 °C, when it is heated to a temperature of 80.00 °C. $\gamma_{water} = 2.1 \times 10^{-4}$ K^{-1}.

6. Explain what is meant by the apparent thermal expansion of a liquid and compare its magnitude with the real thermal expansion of the same liquid under the same conditions.

7. Explain why water expands on freezing.

7.3 Quantity of heat, specific heat capacity and heat capacity

By the end of this section you should be able to:

*   Identify different units of heat energy.
*   Define the terms specific heat capacity and heat capacity.
*   Describe the factors that affect the amount of heat absorbed or liberated by a body.
*   Calculate the amount of heat energy absorbed or liberated by a body using $Q = mc\Delta T$.
*   Calculate the heat capacity of a body.
*   Describe the uses of a calorimeter.
*   Explain the significance of the high specific heat capacity of water.
*   Use the relationship heat lost = heat gained to solve problems involving heat exchange.

What are the units of energy?

As discussed in Unit 4, the scientific unit of energy is the joule (J). You may see another unit of energy called the calorie (cal).

One calorie is the quantity of heat energy required to increase the temperature of 1 g of water by 1 °C. The amount of energy in joules required to increase the temperature of 1 g of water by 1 °C is 4.18 J and so:

*   **1 cal = 4.2 J**

The calorie is less frequently used now but you will see later that its definition is connected to the work we do later in this section.

What is meant by the term specific heat capacity?

If we were heating a substance to raise its temperature, the amount of heat energy required would depend on three things:

1. The **substance** being heated. A given mass of aluminium will require more energy to raise its temperature by 1 K than the same mass of wood.

2. The **mass** of the substance. The greater the mass of the substance, the more heat energy will be required to raise its temperature.

DID YOU KNOW?

The British Thermal Unit is still used in some applications today. It is the quantity of energy needed to raise the temperature of 1 lb of water by 1 °F, which is about 1060 J.

Worked example

Calculate the quantity of heat energy required to heat a 1.00 kg block of iron from 290 K to 320 K. The specific heat capacity of iron is 470 J/kg K.

$Q = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

In this case, m = 1.00 kg, c = 470 J/kg K, ΔT = (320 K – 290 K) = 30 K

Q = 1.00 kg × 470 J/kg K × 30 K
Substitute in known values and complete calculation

Q = 14 100 J = 14.1 kJ *Clearly state the answer with unit*

So, 14.1 kJ of heat energy would be required to increase the temperature of a 1.00 kg iron block by 30 K. Equally, if the 1.00 kg iron block cooled by 30 K, the iron block would have to lose 14.1 kJ of heat energy to the surroundings.

Worked example

Calculate the quantity of heat energy lost from a 580 g sample of water if it cools from 333 K to 278 K. The specific heat capacity of water is 4200 J/kg K.

$Q = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

In this case, m = 0.58 kg, c = 4200 J/kg K, ΔT = (333 K – 278 K) = 55 K

Q = 0.58 kg × 4200 J/kg K × 55 K
Substitute in known values and complete calculation

Q = 13 398 J = 134 kJ *Clearly state the answer with unit*

So, as the 580 g sample of water cools from 333 K to 290 K, it would lose 134 kJ of heat energy to its surroundings.

3. The **temperature rise** required. For a given mass of a particular substance, a large temperature increase will require a larger amount of heat energy than a small increase in temperature.

Each substance has a **specific heat capacity** (c), which is defined as:

- **The heat energy required to raise the temperature of 1 kg of a given substance by 1 K.**

The units of specific heat capacity are J/kg K and Table 7.3 shows the specific heat capacities of some materials. We can see that metals tend to have lower specific heat capacities than non-metals and that water has a notably high value.

Table 7.3 Some different specific heat capacities

Substance	Specific heat capacity (J/kg K)
iron	470
copper	420
brass	380
aluminium	910
water	4200
rubber	1700
glass	670

From the definition of specific heat capacity, the quantity heat energy required (Q) to increase the temperature of a substance is found using the equation below:

- $c = Q / m\Delta T$

This is usually written as:

- $Q = mc\Delta T$

m = mass of substance (kg)

c = specific capacity of substance (J/kg K)

ΔT = change in temperature (K). Remember, a change in temperature of 1 °C is the same as a change in temperature of 1 K. So, in this case °C or K may be used.

Activity 7.6: Specific heat calculations

Calculate the specific heat capacity of a 2.0 kg block of a solid that requires 63 700 J to raise its temperature by 35 K.

A solid has a specific heat capacity of 800 J/kg K. How much heat energy would be released a 250 g sample of this solid if its temperature falls from 310 K to 260 K?

How can we find the specific heat capacity of a substance?

There are several methods we can use to determine the specific heat capacity of a substance, but remember that to calculate this value we will always need to know the **mass** of the substance, the **amount of heat energy** supplied to it as well as its **starting temperature** and **final temperature**. All of these slightly different approaches involve heat exchange. The heat from a hot body is used to warm a colder body. In approaching these heat exchange calculations we use the principle that the heat energy lost from the hot body will equal the heat gained by the cold body.

- **Heat energy lost by hotter body = heat energy gained by colder body**

Sometimes, we try to prevent heat loss to the surroundings using **insulation**. The experimental approach of measuring heat capacities and the heat changes during chemical and physical processes is called **calorimetry**.

Electrical heating

This method can be used to find the specific heat capacity of a solid or a liquid. The diagram in Figure 7.37 shows the apparatus used to find the specific heat capacity of a solid. In this case, the hot body losing the heat energy is the electrical heater.
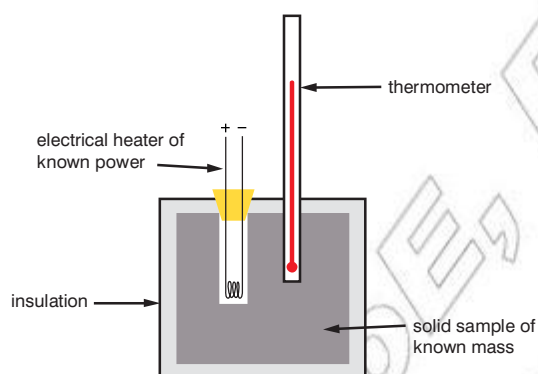

Figure 7.37 Finding the specific heat capacity of a known mass

If we know the power rating of the heater and we know how long the heater is switched on for we can determine the quantity of heat energy supplied to the block. For example, a 100 W electrical heater supplies 100 J of heat energy every second.

We will use the following experimental data to calculate the specific heat capacity of aluminium using this apparatus.

A 100 W electrical heater, running for 5 minutes, warmed a 0.50 kg block. The start temperature of the aluminium block was 20 °C and its final temperature was 85 °C.

Energy supplied = power × time *State principle or equation to be used (from the definition of power)*

$E = P \times t$ *Express in the standard symbols*

$E = 100$ W × (5 minutes × 60) = 100 W × 300 s *Substitute in known values and complete calculation*

Worked example

A 300 g block of brass at 298 K is supplied with 1026 J of energy from an electrical heater. Calculate the final temperature of the brass block after this heating, assuming that there has been no heat loss. The specific heat capacity of brass = 380 J/kg K.

$Q = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

$\Delta T = Q / mc$ *Rearrange equation to make ΔT the subject*

In this case, $m = 0.30$ kg, $c = 380$ J/kg K, $Q = 1026$ J.

$\Delta T = 1026$ J /(0.30 kg × 380 J/kg K) *Substitute in known values and complete calculation*

- $\Delta T = 9$ K *Clearly state the answer with unit*

So, we have found that $\Delta T = 9$ K. As the brass block has been heated, the temperature would have increased and so the final temperature = 298 K + 9 K = 307 K or 34 °C. *Ensure the final temperature is calculated, not just ΔT*

DID YOU KNOW?

Even on his honeymoon in the Swiss Alps, James Joule did not stop work. He tried to show that when water falls through 778 feet, its temperature rises by 1 °F but all the spray got in the way!

---

calorimetry *the experimental approach to measuring heat capacities and heat changes during chemical and physical processes*

final temperature *the temperature of a substance after heating*

insulation *material which does not conduct heat energy and hence can prevent heat loss*

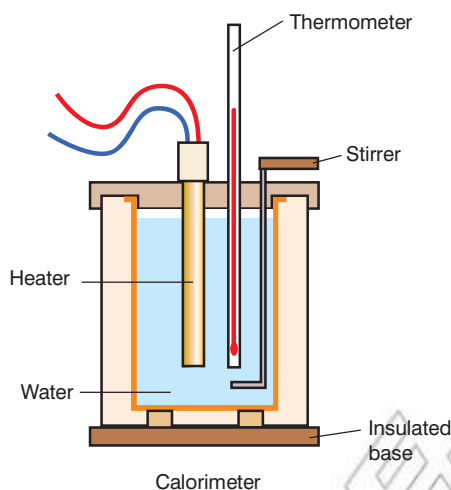**starting temperature** *the temperature of a substance before heating*

Thermometer

Stirrer

Heater

Water

Insulated base

Calorimeter

Figure 7.38 Determining the specific heat capacity of a liquid

DID YOU KNOW?

"*calorimeter*" comes from the Latin *calor*, which means heat.

$E = 30\,000$ J *Clearly state the answer with unit*

Assuming all this energy goes into heating the block

$Q = 30\,000$ J

$Q = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

$c = Q\,/\,m\Delta T$ *Rearrange equation to make c the subject*

In this case, $m = 0.50$ kg, $\Delta T = (85\,°C – 20\,°C = 65\,°C) = 65$ K, $Q = 30\,000$ J.

$c = 30\,000$ J $/(0.50$ kg $\times 65$ K$)$ *Substitute in known values and complete calculation*

$c = 923$ J/kg K *Clearly state the answer with unit*

We have ignored any heat energy supplied to the thermometer and any heat lost to the surroundings, and assumed that the electrical heater is 100% efficient in this calculation.

Electrical heating can also be used to determine the specific heat capacity of a liquid. An insulated container could be used for the liquid and the data obtained will be the same as for the example above. Alternatively, a calorimeter could be used to hold the liquid. A calorimeter is a polished metal can. In this case, the liquid is continuously stirred and we will take into account the heat energy supplied to the calorimeter as well as that supplied to the liquid. The diagram in Figure 7.38 shows the use of a calorimeter to determine the specific heat capacity of water.

We will use some experimental data from this method to calculate the specific heat capacity of water. The important factor to remember here is that some of the heat energy supplied by the heater will warm the calorimeter and stirrer as well as the water.

The electrical heater has a power rating of 200 W. It caused the water to increase in temperature from 25 °C to 74 °C after running for 5 minutes.

Mass of water = 200 g

Mass of aluminium calorimeter and stirrer = 400 g

Specific heat capacity of aluminium = 910 J/kg K

Heat energy supplied by heater (Q_h) = **heat energy received by water (Q_w)** + **heat energy received by aluminium calorimeter (Q_c)**

The water and the stirrer will be in thermal equilibrium and so the temperature change for both will be the same (i.e. $\Delta T = 74\,°C – 25\,°C = 49\,°C = 49$ K).

Heat supplied by heater (Q_h):

$E = P \times t$ *State principle or equation to be used (from the definition of power)*

$E = 200$ W $\times (5$ minutes $\times 60) = 200$ W $\times 300$ s *Substitute in known values and complete calculation*

$E = 60\,000$ J *Clearly state the answer with unit*

$Q_h = 60\,000$ J

Heat energy received by calorimeter and stirrer (Q_c):

$Q_c = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

$Q_c = 0.40$ kg $\times$ 910 J/kg K $\times$ 49 K *Substitute in known values and complete calculation*

$Q_c = 17\,836$ J *Clearly state the answer with unit*

Heat energy received by water (Q_w):

$Q_h = Q_w + Q_c$ *Express the relationships between the energies.*

$60\,000$ J $= Q_w + 17\,836$ J *Substitute in known values*

$Q_w = 60\,000$ J $-$ $17\,836$ J *Rearrange to make Q_w the subject complete calculation*

$Q_w = 42164$ J *Clearly state the answer with unit*

Specific heat capacity of water:

$Q_w = mc\Delta T$ *State principle or equation to be used (from definition of specific heat capacity)*

$c = Q_w / m\Delta T$ *Rearrange equation to make c the subject*

In this case, $m = 0.20$ kg, $\Delta T = 49$ °C, $Q_w = 42\,164$ J.

$c = 42\,164$ J$/(0.20$ kg $\times$ 49 °C) *Substitute in known values and complete calculation*

$c = 4302$ J/kg K *Clearly state the answer with unit*

Once again, this is an experimental value – one major source of error will be heat loss to the surroundings, despite the precautions taken.

Method of mixtures

This method can be adapted to measure the specific heat capacity of a solid or liquid.

The diagram in Figure 7.39 on the next page shows the method used to determine the specific heat capacity of a solid. The solid, of known mass, m_s, is heated in a water bath at 100 °C for at least 5 minutes. The solid is then quickly transferred to the cold water of known mass, m_w, in the calorimeter.

We know that the start temperature of the solid object is 100 °C. Once in the calorimeter, the hot object (in this case a steel bolt) loses some heat energy to the colder water, and the colder calorimeter and stirrer. We stir the water and record the highest temperature on the thermometer.

Activity 7.7: Specific heat capacity calculation

A copper calorimeter and stirrer of mass 350 g contains 250 g of a liquid. A 500 W heater running for 2 minutes and 30 seconds heated this combination of liquid and calorimeter from 20 °C to 88 °C. Calculate the specific heat capacity of this liquid given that the specific heat capacity of copper is 420 J/kg K.
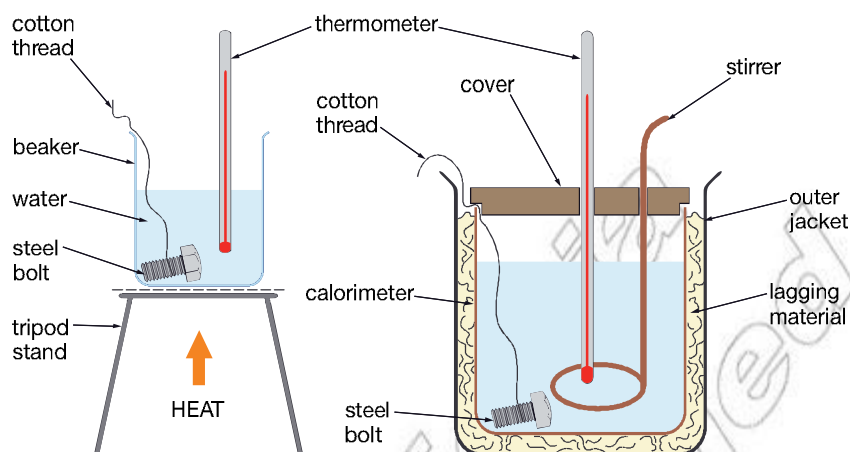
Figure 7.39 *Determining the specific heat capacity of a bolt using the method of mixtures*

In this case, the heat lost by the hot bolt will be equal to the heat gained by the water and the calorimeter/stirrer.

Mass of water = 200 g.
Specific heat capacity of water = 4200 J/kg K.
Mass of copper calorimeter and copper stirrer = 100 g.
Specific heat capacity of copper = 420 J/kg K.

Start temperature of water + calorimeter = 20 °C.
Highest temperature of water after addition of the steel bolt = 25 °C.
Mass of steel bolt = 125 g.

Temperature of steel bolt before cooling = 100 °C.

Heat lost by bolt = heat received by water + heat received by
calorimeter/stirrer
Q_b Q_w Q_c

Heat received by water Q_w:

- $Q_w = mc\Delta T$

    $m = 0.200$ kg

    $c = 4200$ J/kg K

    $\Delta T = 25\ °C - 20\ °C = 5\ °C = 5$ K

- $Q_w = 4200$ J

Heat received by calorimeter/stirrer Q_c:

- $Q_c = mc\Delta T$

    $m = 0.100$ kg

    $c = 420$ J/kg K

    $\Delta T = 25\ °C - 20\ °C = 5\ °C = 5$ K

- $Q_c = 210$ J

Heat lost by bolt Q_b:

- $Q_b = Q_w + Q_c$

- $Q_b = 4200$ J + 210 J = 4410 J

Specific heat capacity of steel bolt c_b:

At the end of the experiment, the bolt, water and the calorimeter and stirrer will be in thermal equilibrium and all be at the same temperature = 25 °C

ΔT = start temperature of bolt – final temperature of bolt = 100 °C – 25 °C = 75 °C = 75 K.

- $Q_b = m \times c_b \times \Delta T$

- $c_b = Q_b / m\Delta T$

- c_b = 4410 J / (0.125 kg × 75 K)

- c_b = 470 J/kg K

This method can be adapted to find the specific heat capacity of a liquid by adding a hot solid of known specific heat capacity to the liquid sample or even by pouring a hot liquid into a cooler one.

What is the heat capacity of a body?

We have seen lots of examples of specific heat capacity. The word *specific* here tells us that this is the heat energy required to increase the temperature of 1 kg of a substance by 1 K. In other words, this is specific to *1 kg* of the substance.

The **heat capacity** of a body is the defined as the energy required to raise the temperature of the given body by 1 K; the mass of the body is not considered, only the energy required to raise its temperature by 1 K.

- **heat capacity** $= Q / \Delta T$

Let us look again at the calculation to determine the specific heat capacity of the steel bolt.

The experimental data shows us that the steel bolt lost 4410 J and its temperature fell by 75 K, or, to increase its temperature by 75 K, 4410 J of heat energy would be required.

So, if 4410 J of energy causes a 75 K rise, the heat capacity would be:

- heat capacity $= Q / \Delta T$

- heat capacity $=$ 4410 J / 75 K

- heat capacity $=$ 58.8 J/K.

Note that the units do not include a mass term.

The specific heat capacity of steel is 470 J/kg K and if the heat capacity of a body is known, the specific heat capacity of the material of which it is made can be found provided the mass of the body is known.

- **Specific heat capacity = heat capacity of body / mass of body**

Let's try this with the steel bolt used in the last specific heat capacity experiment.

So, for the steel bolt of mass 0.125 kg:

Activity 7.8: Heat capacity of a ball

A plastic ball experiences a temperature rise of 10 K when 600 J of heat energy are supplied to it. Calculate the heat capacity of the ball. What else would you need to know in order to calculate the specific heat capacity of the ball?

Figure 7.40 Water's high specific heat capacity makes it useful for cooling systems.

- specific heat capacity = heat capacity of body / mass of body
- specific heat capacity = 58.8 J/K / 0.125 kg
- specific heat capacity = 470 J/kg K

We will now try another example:

A solid block requires 3000 J of heat energy to increase its temperature by 60 K. Calculate the block's heat capacity and use this value to calculate its specific heat capacity if the block has a mass of 50 g.

- heat capacity = $Q / \Delta T$
- heat capacity = 3000 J / 60 K
- heat capacity = 50 J/K
- specific heat capacity = heat capacity of body / mass of body
- specific heat capacity = 50 J/K / 0.05 kg
- specific heat capacity = 1000 J/kg K

Why is the high specific heat capacity of water so important?

If you look back at the table of specific heat capacities of different substances, you will see that water has an especially high value. We should think about this further and see if it is important.

If a substance has a high specific heat capacity, it means that a large amount of heat energy is required to bring about a rise in temperature of 1 kg water by 1K. This is important when we remember that water is widely used in industry and in internal combustion engines for cooling.

If a liquid with a low specific heat capacity was used for cooling purposes, a given mass of this liquid would receive very little heat energy before its temperature increased to its boiling point, at which stage it would no longer act as a coolant.

So, the fact that a given mass of water will receive a large amount of heat energy compared to other liquids before it boils makes it very useful for cooling.

The reverse is also true. A large mass of hot water contains a very large amount of energy. This can then be pumped around the house and as the water cools is transfers this heat energy to rooms inside the house.

Summary

In this section you have learnt that:

- The specific heat capacity of a substance is the heat energy required to raise the temperature of 1 kg of a given substance by 1 K.

- The heat capacity of a body is the heat energy required to raise the temperature of the body by 1 K.

- The factors that affect the amount of heat absorbed or liberated by a body are the temperature change, the mass of the body and the material making up the body.

- The amount of heat energy absorbed or liberated by a body can be calculated using $Q = mc\Delta T$.

- The high specific heat capacity of water is significant as water is used for cooling.

- A calorimeter is used in specific heat capacity and heat capacity experiments.

Review questions

1. Define the term "specific heat capacity of water".

2. A metal bar of mass 100 g is warmed from 20 °C to 80 °C. How much heat is absorbed by the metal bar if the specific heat capacity of this metal is 450 J/kg K.

3. In an experiment to calculate the specific heat capacity of a metal, the following results were obtained:

   Mass of metal = 300 g.

   Start temperature = 20 °C.

   End temperature = 75 °C.

   Power rating of electrical heater = 100 W.

   Time of heating = 150 s.

   Use this data to calculate the specific heat capacity of the metal.

4. A hot metal block is placed into 50 g of water in an insulated container. The water increases in temperature from 20 °C to 32 °C. The specific heat capacity of water is 4200 J/kg K. Calculate the quantity of heat energy supplied to the water from the metal block.

7.4 Changes of state

By the end of this section you should be able to:

- Define the terms latent heat, latent heat of fusion and latent heat of vaporisation.
- Solve problems involving change of state.

Heating and cooling curves

If we heat a solid, its particles gain energy and begin to vibrate faster and move further apart as its temperature increases. This continues until the solid melts. Even though the solid is continuously heated as it melts, its temperature will not increase until the entire solid has **melted**.

If the heat energy the substance gains is not used to increase the average kinetic energy of its particles (the temperature does not change) what is it being used for? We can see that the same effect occurs when a liquid boils. As the liquid is being heated at its boiling point, the temperature does not increase until all of the liquid has boiled.

The graph in Figure 7.42 shows how the temperature of a solid (A) varies with time as it is heated until melts (B) and finally forms a liquid (C). This is called a **heating curve**. The liquid is heated until it boils (D) until all of the liquid changes state into a gas (E), which continues to increase in temperature as it is heated.
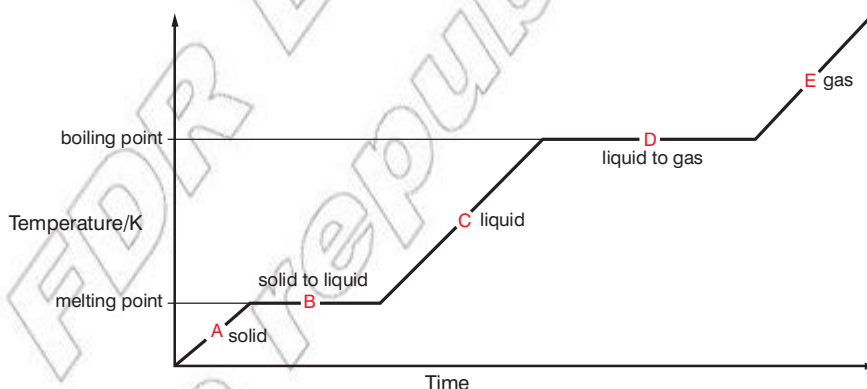

Figure 7.42 This graph shows how the temperature of water changes as it is continuously heated.

We cannot link the heat energy being absorbed during melting and boiling with an increase in temperature and so the heat energy appears to be hidden or latent.

If latent heat is not being used to increase the kinetic energy of the particles of a substance, what is it being used for? During a change in state, the forces of attraction holding the particles together have to be broken. This process requires energy and so, as a solid melts or a liquid boils, the heat supplied is used to separate the particles rather than to increase their kinetic energy. Consequently, the temperature of the substance does not change during a change in state. This is true for melting, boiling as well as condensing and freezing.

A similar shape is seen when the **cooling curve** of a substance is examined (Figure 7.43). When changes of state occur, the temperature remains constant as only potential energy is being lost as forces of attraction act between the particles again.

Figure 7.41 No matter how long you heat it the temperature of boiling water will not go over 100 °C.

DID YOU KNOW?

The term **phase** is sometimes used instead of **state** – i.e. at room temperature and pressure water is in a liquid phase.

KEY WORDS

cooling curve *a graph showing the temperature of a substance against time as it loses heat energy and changes state*

heating curve *a graph showing the temperature of a substance against time as heat energy is applied and it changes state*

melted *when a substance has changed from a solid to a liquid state*

phase *the distinct form of a substance under different conditions e.g. solid, liquid, gas*
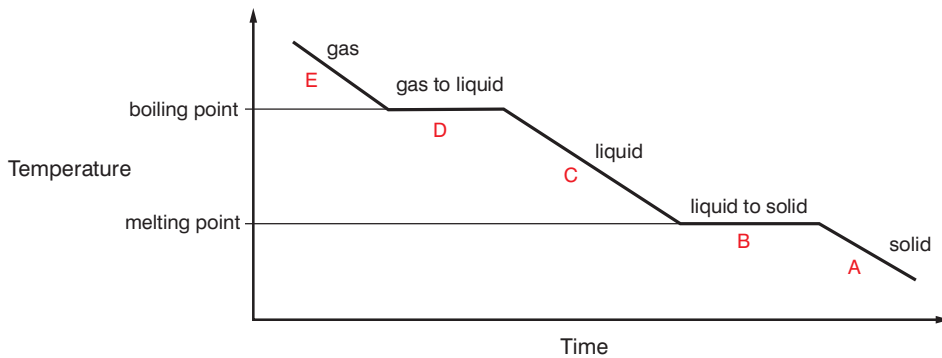
Figure 7.43 *A cooling curve for a gas cooling to eventually become a solid.*

Definition of specific latent heat

The specific latent heat (*L*) of a substance is defined as:

- **The quantity of heat energy required to change 1 kg of a substance from one state to another, at constant temperature.**

Specific latent heat has the units of J/kg. Notice again that we use the term 'specific' as this quantity is 'specific' for 1 kg of a substance.

In terms of an equation we have:

- $L = Q / m$

  L = specific latent heat in J/kg

  Q = energy required in J

  m = mass in kg

This is usually written as:

- $Q = mL$

However, there are two changes of state to consider, liquid to gas and solid to liquid. We use two different versions of latent heat, the **latent heat of fusion** (melting) and **latent heat of vaporisation** (boiling).

Specific latent heat of fusion (*L$_f$*)

This is the quantity of heat energy required to change 1 kg of a substance from a solid to a liquid at constant temperature.

Specific latent heat of vaporisation (*L$_v$*)

This is the quantity of heat energy required to change 1 kg of a substance from a liquid to a gas at constant temperature.

Table 7.4 shows the values of the specific latent heat of fusion and specific latent heat of vaporisation for some elements and compounds.

Activity 7.9: A cooling curve

Draw the shape of a cooling curve, starting at 90 °C, for a substance that has a boiling point of 83 °C and a melting point of 40 °C. Continue the cooling curve until the temperature reaches 25 °C.

KEY WORDS

state *the distinct form of a substance under different conditions e.g. solid, liquid, gas*

latent heat of fusion *the amount of heat energy required to change the state of a substance from a solid to a liquid at constant temperature*

latent heat of vaporisation *the amount of heat energy required to change the state of a substance from a liquid to a gas at constant temperature*

Worked example

1. Calculate the heat energy required to melt 1 kg of copper at its melting point.

   Heat change absorbed on melting:

   $Q = m \times L_f$ *State principle or equation to be used (from definition of specific latent heat)*

   $Q = 1 \text{ kg} \times 209\ 000 \text{ J/kg}$
   Substitute in known values and complete calculation

   $Q = 209\ 000 \text{ J} = 209 \text{ kJ}$
   Clearly state the answer with unit

2. Calculate the mass of water that changes state if the water is at its boiling point and 500 kJ of energy is supplied.

   $Q = m \times L_v$ *State principle or equation to be used (from definition of specific latent heat)*

   $m = Q / L_v$ *Rearrange equation to make m the subject*

   $m = 500\ 000 \text{ J} / 2\ 500\ 000 \text{ J/kg}$
   Substitute in known values and complete calculation

   $m = 0.2 \text{ kg}$ *Clearly state the answer with unit*

3. Calculate the heat energy liberated when 0.025 kg of aluminium freezes at its freezing point.

   Heat change given out on freezing:

   $Q = m \times L_f$ *State principle or equation to be used (from definition of specific latent heat)*

   $Q = 0.025 \text{ kg} \times 390\ 000 \text{ J/kg}$
   Substitute in known values and complete calculation

   $Q = 9750 \text{ J} = 9.75 \text{ kJ}$ *Clearly state the answer with unit*

   In this case, this is the energy given out to the surroundings as the aluminium freezes.

Table 7.4 Some specific latent heats of fusion and vaporisation

Substance	Specific latent heat of fusion, L_f (J/kg)	Specific latent heat of vaporisation, L_v (J/kg)
aluminium	390 000	10 900 000
copper	209 000	4 730 000
gold	63 700	1 645 000
iron	245 000	6 080 000
water	334 000	2 500 000

Looking carefully at Table 7.4 we can see that the specific latent heat of vaporisation is always much higher than the specific latent heat of fusion. This means it takes a great deal more energy to turn a liquid to a gas than it does to turn a solid into a liquid. This is because when a substance changes from a liquid to a gas the bonds between all the molecules have to be broken apart, whereas the particles in a liquid remain bonded together.

The melting point of aluminium is 660 °C. So, at 660 °C, 1 kg of solid aluminium would require 390 000 J of heat energy to change its state into a liquid. This also means that 390 000 J of heat energy would be given out if 1 kg of liquid aluminium at 660 °C changed state into a solid.

Now we will work through some example calculations, using the specific latent heat data in Table 7.4.

Activity 7.10: Changes of state

Calculate the heat energy required to boil 100 g of aluminium at its boiling point. L_v (aluminium = 10 900 000 J/kg).

Calculate the heat energy liberated when 2 kg of water turns to a solid at its melting point. L_f (water) = 334 000 J/kg.

Now that we can use specific heat capacities and specific latent heats, we can calculate the heat energy required when substances are heated, taking into account changes in state.

- **Total energy required = energy required to increase temperature + energy required to change state**

- $Q_{total} = mc\Delta T + mL$

For example, we will now calculate the heat energy required to increase the temperature of 50 g of water from 25 °C to 125 °C.

Specific heat capacity of water = 4200 J/kg K.

Specific heat capacity of steam = 2080 J/kg K.
Specific latent heat of vaporisation of water = 2 501 000 J/kg.

We will need to calculate the heat energy required for this change in three stages:

1. Heat energy required to heat 50 g of water from 25 °C to 100 °C (ΔT = 75 °C = 75 K):

   - $Q = mc\Delta T$
   - $Q = 0.05$ kg $\times$ 4200 J/kg K $\times$ 75 K
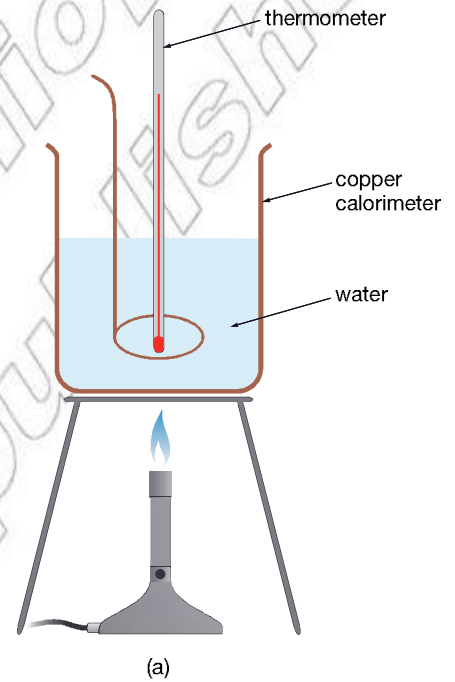   - $Q_1 = 15\,750$ J

2. Heat energy required to boil 50 g of water at 100 °C:

   - $Q = mL_v$
   - $Q = 0.05$ kg $\times$ 2 501 000 J/kg
   - $Q_2 = 125\,050$ J

3. Heat energy required to heat 50 g of steam from 100 °C to 125 °C (ΔT = 25 °C = 25 K):

   - $Q = mc\Delta T$
   - $Q = 0.05$ kg $\times$ 2080 J/kg K $\times$ 25 K
   - $Q_3 = 2600$ J.

   So, the total amount of heat energy required for this process:

   - $Q = Q_1 + Q_2 + Q_3$
   - $Q = 15\,750$ J + 125 050 J + 2600 J
   - $Q = 143\,400$ J = 143.4 kJ

Experiment to determine the specific latent heat of fusion of ice

The apparatus in Figure 7.44 can be used to determine the specific latent heat of fusion, L_f, of ice.

The copper calorimeter and stirrer is weighed before being half filled with water. The mass of the water present in the calorimeter is then determined before the water is heated to at least 10 °C above room temperature. Small quantities of ice are then added to the water, while stirring, until the temperature is below room temperature and all the ice has melted. The mass of the calorimeter, stirrer and water is then determined to find out the mass of ice added.

We will use the experimental data obtained using this method to determine the specific latent heat of fusion of ice.

Mass of calorimeter = 0.15 kg.
Mass of water = 2.00 kg.
Mass of ice added = 0.60 kg.
Start temperature of ice = –10 °C.
Start temperature of water = 49 °C.
Final temperature of water = 20 °C.
Specific heat capacity of water = 4200 J/kg K.
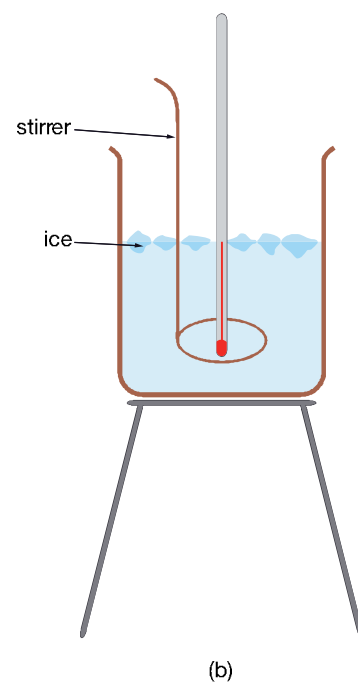

(a)

(b)

Figure 7.44 A simple experiment to determine the specific latent heat of fusion of ice.

Specific heat capacity of ice = 2100 J/kg K.
Specific heat capacity of copper = 420 J/kg K.

We will use the principle here that the heat energy lost from the water and the calorimeter will go to warming and melting the ice and then warming the cold water produced when the ice melts.

Heat energy absorbed by ice = Heat energy liberated from
and cold water   calorimeter/stirrer and water and water

1. **Heat energy lost by calorimeter/stirrer and water:**

   - ΔT = 49 °C – 20 °C = 29 °C = 29 K

   Heat energy lost by calorimeter:

   - $Q_{\text{lost calorimeter}} = m_{\text{calorimeter}} c_{\text{calorimeter}} \Delta T$
   - $Q_{\text{lost calorimeter}}$ = 0.15 kg × 420 J/kg K × 29 K = 1827 J

   Heat energy lost by water:

   - $Q_{\text{lost water}} = m_{\text{water}} c_{\text{water}} \Delta T$
   - $Q_{\text{lost water}}$ = 2.00 kg × 4200 J/kg K × 29 K = 243 600 J

   Total heat energy lost by calorimeter/stirrer and water = 243 600 + 1827 = 245 427 J.

2. **Heat energy absorbed by ice and water in warming from –10 °C to 20 °C:**

   Heat energy require to warm ice from –10 °C to 0 °C (ΔT = 0 °C – –10 °C = 10 °C = 10 K).

   - $Q_{\text{ice} (-10-0)} = m_{\text{ice}} c_{\text{ice}} \Delta T$
   - $Q_{\text{ice} (-10-0)}$ = 0.60 kg × 2100 J/kg K × 10 K = 12 600 J

   Heat energy required to melt 0.60 kg of ice:

   - $\mathbf{Q_{\text{melt ice}} = mL_f}$
   - $\mathbf{Q_{\text{melt ice}}}$ = **0.60 kg** $\times L_f$

   Heat energy require to warm cold water from 0 °C to 20 °C (ΔT = 20 °C – 0 °C = 20 °C = 20 K).

   - $Q_{\text{water} (0-20)} = m_{\text{ice}} c_{\text{water}} \Delta T$
   - $Q_{\text{water} (0-20)}$ = 0.60 kg × 4200 J/kg K × 20 K = 50 400 J

   Heat energy absorbed by ice = Heat energy liberated from
   and cold water   calorimeter/stirrer and water

   12 600 J + $0.6L_f$ + 50 400 J = 245 427 J

   - $0.6L_f$ = 182 427 J
   - L_f = 182 427 J / 0.6
   - L_f = 304 045 J/kg = 304 kJ/kg

Summary

In this section you have learnt that:

- The terms specific latent heat of fusion and specific latent heat of vaporisation relate to the energy required to melt and vaporise 1 kg of a substance at constant temperature.

- It is possible to calculate the energy (Q) required or liberated on a change of state for a given mass, m, of a substance using the equation $Q = mL_f$ or $Q = mL_v$.

Review questions

1. Calculate the heat energy required to melt 10 g of copper at its melting point.

   (L_f for copper = 209 000 J/kg).

2. Calculate the heat energy required to melt 1.2 kg of gold at its melting point.

   (L_f for gold = 63 700 J/kg).

3. Calculate the heat energy liberated when 75 g of iron freezes at its freezing point.

   (L_f for iron = 245 000 J/kg).

4. Define the term specific latent heat of fusion of magnesium.

5. Calculate the heat energy required to increase the temperature of 0.1 kg of water from 10 °C to 150 °C.

   Specific heat capacity of water = 4200 J/kg K.
   Specific heat capacity of steam = 2080 J/kg K.
   Specific latent heat of vaporisation of water = 2 500 000 J/kg.

End of unit questions

1. Write a paragraph explaining the difference between the heat energy in a substance and the substances' temperature.

2. Explain, with reference to the appropriate laws of thermodynamics and particle movement, what happens when a cold object is in thermal contact with hot object.

3. Calculate the increase in length of an iron pipeline that is 30.00 m long at 20 °C when it is warmed to 45 °C. α_{iron} = 1.1 × 10^{-5} K^{-1}.

4. Calculate the increase in volume of ethanol that has a volume of 2.5 × 10^{-4} m^3 at 25 °C when it is warmed to 45 °C. Explain why the apparent expansion will be less than this calculated real expansion. $\gamma_{ethanol}$ = 75 × 10^{-5} K^{-1}.

5. In an experiment to calculate the specific heat capacity of a metal, the following data were obtained. Use the data to calculate the specific heat capacity of the metal.

Mass of metal = 200 g.
Start temperature = 20 °C.
End temperature = 105 °C.
Heat energy supplied by electrical heating = 2000 J.

6. A metal block increases in temperature from 15 °C to 60 °C when supplied with 13 500 J of heat energy.

   a) Calculate the heat capacity of the metal.

   b) Calculate the specific heat capacity of the metal, if this sample has a mass of 0.75 kg.

7. 20 g of water at 42 °C was placed in a well-insulated copper calorimeter with a mass of 27 g at a temperature of 20 °C. Use the specific heat capacities of water (4200 J/kg K) and copper (420 J/kg K) to determine the final temperature of the water.

8. Calculate the heat energy required to increase the temperature of 10.0 kg of water from 25 °C to 115 °C.

   Specific heat capacity of water = 4200 J/kg K.
   Specific heat capacity of steam = 2080 J/kg K.
   Specific latent heat of vaporisation of water = 2 500 000 J/kg.

9. Sketch a cooling curve for bromine as bromine vapour is cooled from 100 °C to –20 °C. Bromine has a melting point = –7 °C and a boiling point of 59 °C. Mark clearly on your graph the melting and boiling point.

10. In an experiment to determine the latent heat of fusion of ice, 0.5 kg of ice at –5 °C was placed into 1.5 kg of water in a copper calorimeter of mass (including stirrer) of 0.2 kg with both water and calorimeter at 61 °C. The final temperature, when all the ice had melted, was 25.0 °C. Use the data to calculate the latent heat of fusion of ice.

    Mass of calorimeter = 0.20 kg.
    Mass of water = 1.50 kg.
    Mass of ice added = 0.50 kg.
    Start temperature of ice = –5.0 °C.
    Start temperature of water = 61 °C.
    Final temperature of water = 25.0 °C.
    Specific heat capacity of water = 4200 J/kg K.
    Specific heat capacity of ice = 2100 J/kg K.
    Specific heat capacity of copper = 420 J/kg K.